# *Signal Propagation Mathematics*

## *Colorado State University*

## *Math Day*

**1st Place**
**Original Mathematics Research**

**Poster Board Summary**

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# I. Introduction

## A. Overview

This experiment compares the propagation pattern of a horizontal dipole antenna at different heights.

1. I construct a dipole antenna and use a free software package, MMANA-GAL, to model its radiation pattern.
2. I will transmit a series of short beacon signals using a transceiver. Other radio operators from around the world that pick up these signals will automatically report the contact to the Weak Signal Propagation Reporter (WSPR), an open source reverse-beacon network.
3. I will analyze and compare the signal reports generated for each antenna height against each other to determine which sends signals further.

My independent variable is the antenna height, and my dependent variable is the distance of the reporting station.

## B. Hypothesis

Spots from the higher antenna would go further because the higher the antenna, the lower the take-off angle that the maximal strength signal radiates from the antenna, and therefore the signal will travel further as it enters the ionosphere at a greater angle of incidence and at a greater distance from the signal's origin.

# II. Process & Procedures

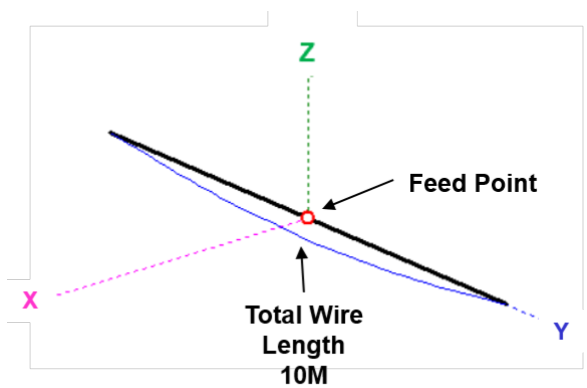
## A. Construct

I constructed a 20m, center-fed dipole antenna. The antenna was made of two equal lengths of 14 gauge copper wire which were approximately ¼ λ in length each. These elements are connected in the center by coaxial cable, which connects to my transceiver. My antenna is tied between two trees, and I am able to move it up and down.

## B. Model

I modeled a 20m dipole antenna in MMANA-GAL antenna modelling software. This free software allowed me to generate propagation patterns of the antenna's signal from an overhead, elevation and 3D viewpoint. From this model, I was able to estimate the angle of maximum directional gain of an antenna's signal at different heights, which helped me estimate which configuration will go further.

**Figure 1: 3D view – dipole antenna**




*Source: Antenna schematic created by Maxwell Moran, W3LLA using MMANA-GAL Software*

## C. Transmit

I ran a series of ≈2 minute long WSPR transmission sequences on a HF transceiver on the 20m band (14MHz) using a dipole antenna at two different heights, one at 1/3 wavelength (λ) above the ground and the other at 1/8 λ. These one-way, beacon-like signals are very narrow in bandwidth and only include my call sign, location, and power in decibels (dB) (e.g. W3LLA DN70 37). After each cycle, signal reports from other radio operators (spotters), are automatically posted to the WSPR database for analysis.

The QRP Labs QCX is an example of a cheap ($49) transceiver which I constructed and hand soldered.  In its intended design, the QCX is only a WSPR transmitter, however, I designed and published a modification to allow the QCX to receive signals making it a pure transceiver.  QRP Labs published this modification on their website, which is located here: *http://www.qrp-labs.com/qcx/qcxmods/qcxwspr*

**Figure 2: QRP Labs, QCX Transceiver**


*Source: W3LLA*

# D. Analyze

I analyzed the data from the WSPR database using a spreadsheet (e.g. LibreCalc or Excel) to see if there was a noticeable difference in spot distance if I changed the height of my antenna.

The WSPR database lists the distance and the directional azimuth from my position to all of the radio operators who heard my beacon (spots).  To make this analysis visually meaningful, I plotted the spots on a Cartesian plane rather than relying on WSPR's mapping function.  To do this, I converted the WSPR spot data (provided in polar coordinates) to rectangular coordinates by using trigonometry, and I plotted the points in a Excel using a scatter chart.

# III. Background

## A. Radio Signals

Radio signals (RF signals/electromagnetic radiation) travel at the speed of light and in a wavelike pattern. Radio operators refer to the frequency in Hz and the wavelength in meters interchangeably. High Frequency (HF) signals are in the 3MHz to 30 MHz frequency range.

**Formulas and equations:**

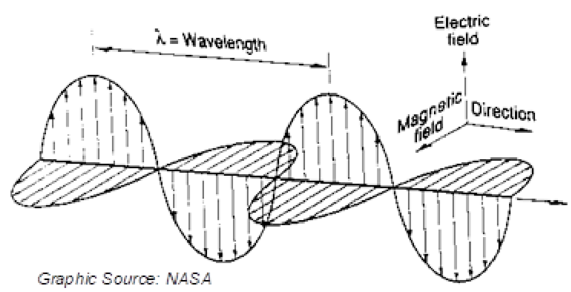
$c = \lambda \cdot f$

$\lambda = c / f$

$f = c / \lambda$

***Where:***

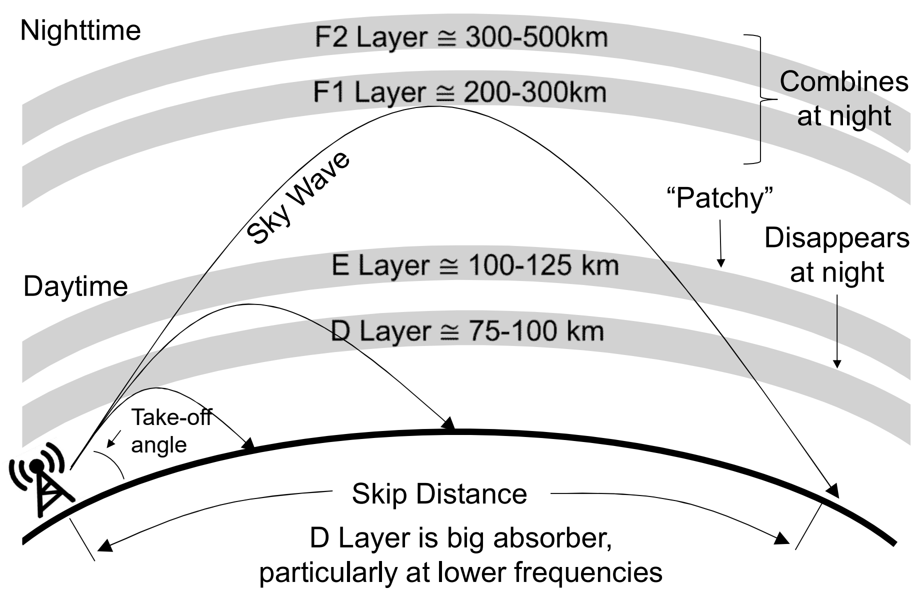
*c = Speed of light ≅ 300M m/s*
*λ = wavelength in meters*
*f = frequency in MHz*




## B. Sky Wave Propagation

HF signals travel great distances because radio signals can be refracted back to Earth by the electrically charged layer of the upper atmosphere called the ionosphere.[1]

**Figure 3: Illustrated ionospheric layers (Source: W3LLA)**



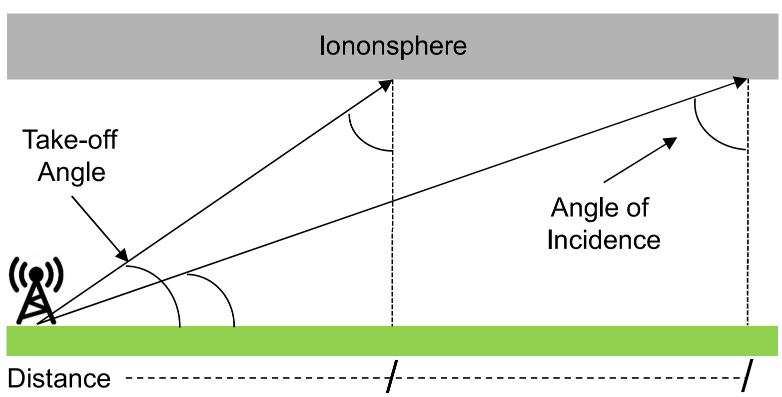
*Source: W3LLA*

# C. Sky Wave – Key Determinants

Radio wave propagation using the ionosphere depends upon a number of interrelated factors, <u>some</u> of these include:

- **Ionospheric Conditions** – The ionosphere is greatly impacted by the 11 year sunspot cycle, solar flares, and daytime (diurnal) solar radiation.
- **Transmission frequency** - At any given time and location, there is an ever changing maximum usable frequency (MUF) above which signals pass through into space, and there is a lowest usable frequency (LUF) below which signals are absorbed. Signal attenuation is the inverse square of the frequency. Doubling the frequency reduces the level of attenuation by a factor of four.
- **Take-off angle** - 90° is directly overhead and 0° is directed towards the horizon. As a rule of thumb, the lower the take-off angle that a signal is sent, the further it will travel because it enters the ionosphere at a greater angle of incidence and at a greater distance from the signal's origin than if the signal was directed directly overhead.

**Figure 4: Take-off angles**




*Source: W3LLA*

# D. Take-off Angles and the Ionization Layers – A Simplified Model

In a simplified model[2], if you assume a signal is sent at a zero degree take-off angle to the earth, a line tangential to the earth is formed. From this we can estimate the maximum distance a signal can travel in one hop if we know the height of the ionospheric refraction.

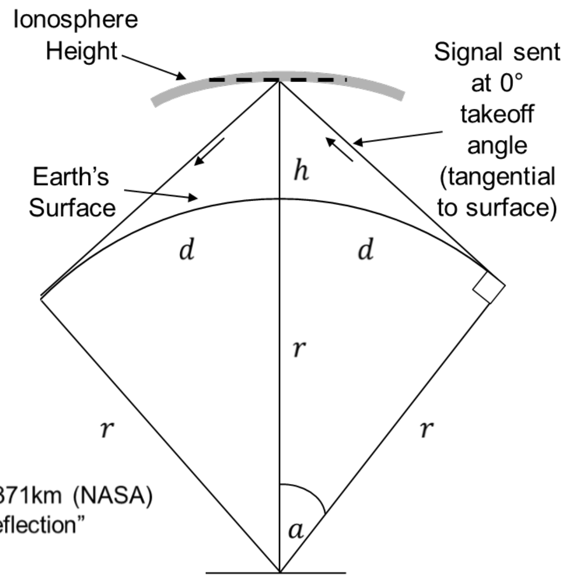
**Figure 5: Simplified ionospheric model**

$$\cos a = \frac{r}{r+h}$$

$$a\,rad = \frac{\pi}{180}\left[\cos^{-1}\left(\frac{r}{r+h}\right)\right]$$

$$d = r \times a\,rad$$

$$\therefore\ Total\ Distance = 2d$$




<u>Where:</u>
$r$ = Volumetric mean radius of Earth ≅ 6,371km (NASA)
$h$ = Assumed height of the ionospheric "reflection"
$d$ = ½ the skip distance in km
$\cos^{-1}$ is expressed in degrees

<u>Example:</u>

$$\cos a = \frac{6{,}371km}{6{,}371km + 75km} = 0.988$$

$$a\,rad = \frac{\pi}{180}\left[\cos^{-1}\left(\frac{6{,}371km}{6{,}371km+75km}\right)\right] = 0.153 \text{ rad (or } 8.749°)$$

$$d = r \times a\,rad = 6{,}371\text{km} \times 0.153 = 973 \text{ km}$$

$$\therefore\ Total\ Distance = 2d = \mathbf{1{,}946km}$$

Ionospheric refraction at the lower D layer of approximately 75km produces a single skip distance of approximately 2,000km while upper F Layer refraction generates distances of approximately 4,402km. The lower the take-off angle and higher the refraction height, the greatest single hop distance.

| Ionosphere Height (km) | Single Skip Distance(km) |
|---|---|
| 75 | 1,946 |
| 100 | 2,243 |
| 150 | 2,738 |
| 200 | 3,152 |
| 300 | 3,836 |
| 400 | 4,402 |

# IV. Antenna Modeling

## A. Antenna Modeling Background

One purpose of antenna modeling is to illustrate the radiation pattern of a specific configuration. Signal strength/weakness in any direction is expressed as a gain/loss compared to an antenna floating in free space, or an isotropic antenna. This gain is expressed in decibels (dBi), a base ten logarithm.

For example, a gain of 3 dBi has 2 times more gain than an isotropic antenna while a gain of 10 dBi has a gain of 10 times.

**Figure 6: Decibel relation to power ratio**

Gain or Loss (expressed in dB)

$$x = 10 * \log_{10} \frac{p2}{p1}$$

Power Ratio

$$\frac{p2}{p1} = 10^{\left(\frac{x}{10}\right)}$$

Where:

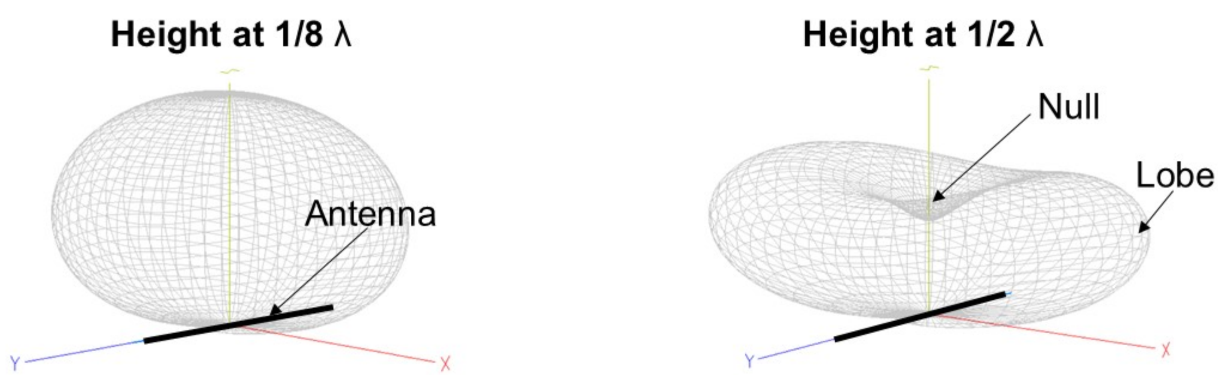
$p1$ is the reference to which $p2$ is being compared

$x$ expressed in decibels

| Gain or Loss (dB) | Power Ratio (multiple) |
|---|---|
| +30 | $10^{3}$ |
| +20 | $10^{2}$ |
| +10 | 10 |
| +6 | 4 |
| +3 | 2 |
| 0 | 1 |
| -3 | ½ |
| -6 | ¼ |
| -10 | $10^{-1}$ |
| -20 | $10^{-2}$ |
| -30 | $10^{-3}$ |

In a dipole antenna the signal propagates broadside to the wire element and the radiation patterns create lobes and nulls of varying signal intensity as the antenna is raised or lowered.

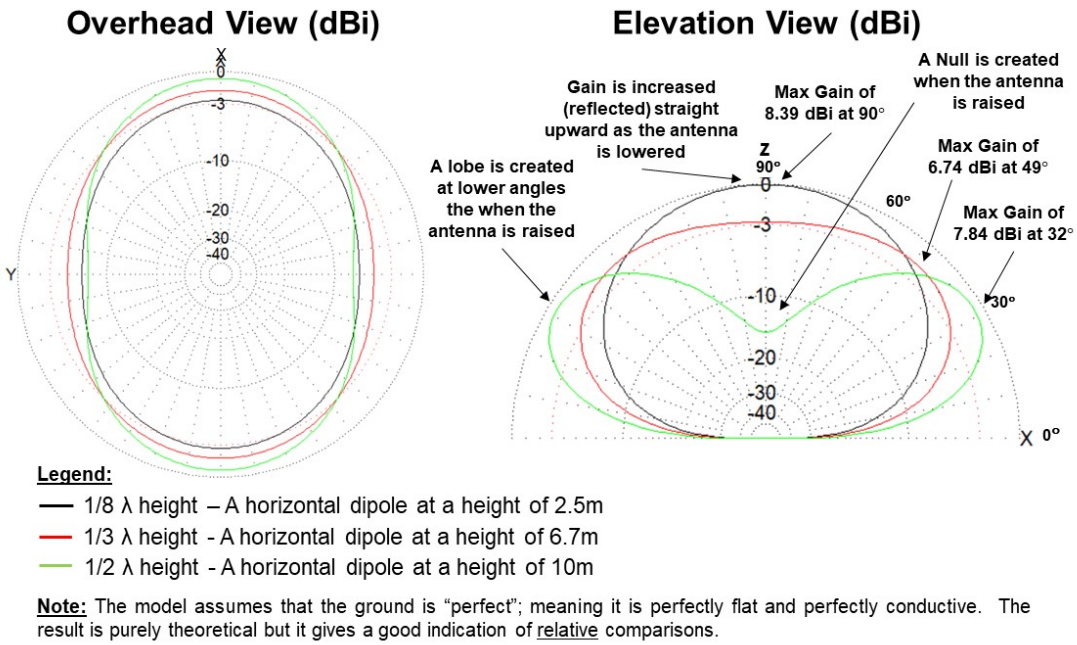
**Figure 7: 3D view of the propagation pattern of a dipole at different heights**




*Source:W3LLA & MMANA-GAL*

I modelled three configurations using the MMANA-GAL software. The figures below are the far field charts which compare the field strength, expressed in dBi, of a signal at some distant point relative to an isotropic antenna (a purely theoretical and omnidirectional antenna in free space).

**Figure 8: Antenna radiation patterns at different heights**




*Source:W3LLA & MMANA-GAL*

The 1/3 λ antenna is 4.7x stronger than an isotropic antenna at an elevation of 49° and the 1/8 λ antenna is 6.9x stronger at 90° (directly above).

| Height | $^1/_8$ λ | $^1/_3$ λ | $^1/_2$ λ |
|---|---|---|---|
| Max Gain | 8.39 dBi | 6.74 dBi | 7.84 dBi |
| Elevation Angle of Max Gain | 90° | 49° | 32° |
| Gain(x) $\frac{p2}{p1} = 10^{(\frac{x}{10})}$ | 6.9x | 4.7x | 6.1x |

You will notice from the far field figures above the creation of lobes and nulls in the radiated pattern as an antenna is raised or lowered. "These formations arise from the reflection of the antenna's radiated energy by the ground….the actual radiation pattern is composed of energy received directly from the antenna and energy that has been reflected from the ground. The direct and reflected signals take different amounts of time to get to the receiving station so they can add together, cancel each other out, or any combination in between."[3]

"The higher the horizontal antenna, the lower is the lowest lobe of the pattern. As a very general rule of thumb, the higher an HF antenna can be placed above the ground, the farther it will provide effective communications because of the resulting lower radiation angle. This is true for any horizontal antenna over real and well as theoretically perfect ground." [4]

# V. WSPR Field Test

## A. WSPR Background

WSPR (pronounced "whisper") stands for "Weak Signal Propagation Reporter" and was created by Dr. Joe Taylor, K1JT, Nobel Laureate (Physics, 1993). This program is designed for sending and receiving low-power transmissions to test propagation paths mainly on the High Frequency (3-30 MHz) bands.

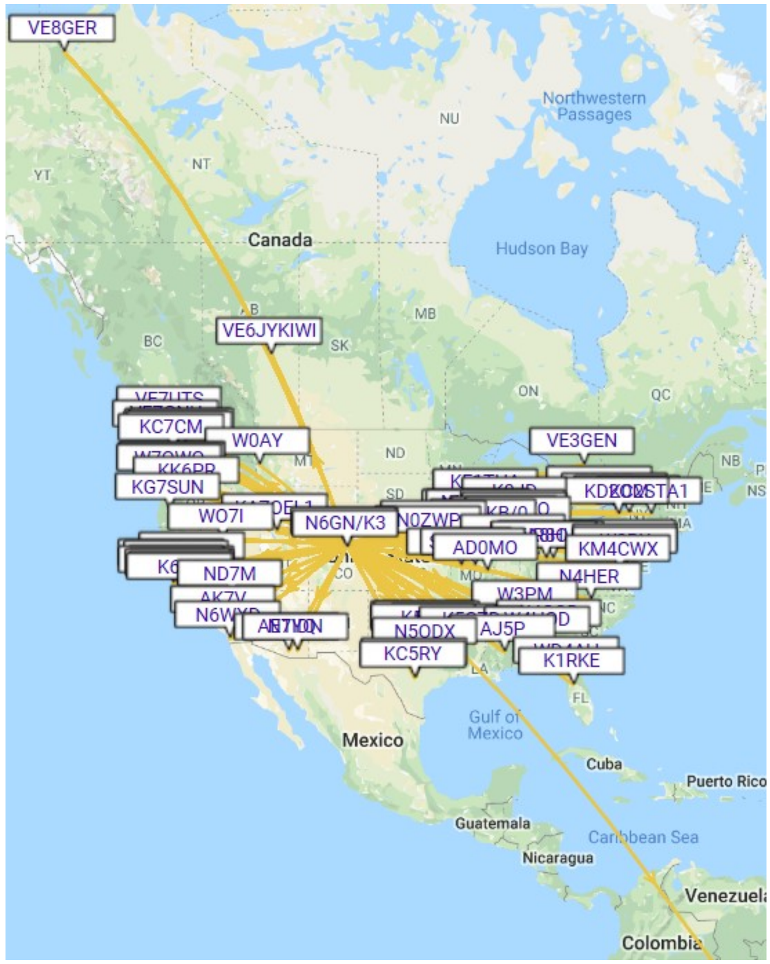
- Designed for one way, minimal contact (call sign, location, & power level).
- Each WSPR transmission cycle lasts one minute and 50 seconds and at the end of each cycle, the signal reports are posted to the internet database.
- WSPR transmissions are a low power, very narrow bandwidth mode (6 Hz). I generated these transmissions using only 5 Watts.

On June 15, 2019, I conducted a series of WSPR transmissions using a 20m dipole at two heights, 1/3 λ and 1/8 λ. I chose these heights because the highest I could make my antenna was only 22 feet (1/3 λ). I ran my transmissions on the 20m (14 MHz) band.

I ran my transmissions for approximately 45 minutes before noon and 45 minutes after noon with a brief pause in the middle to change the height. In total, I ran a series of 21, two minute cycles at each height, totalling 42 minutes of total transmission time over 90 minutes.

The WSPR database lists the distance and the directional azimuth from my position to all the radio operators who heard my beacon (spots). I downloaded the WSPR data into Excel, and I plotted the spots on a Cartesian plane (scatter chart) by converting the WSPR spot data (provided in polar coordinates) to rectangular coordinates by using trigonometry. I also inverted the x and y axis to fix the 0° mark to be on the top (i.e. the north position).

**Figure 9: Total WSPR spots 20m band**




*Source: http://wspr.vk7jj.com/*

**Table 1: Example spreadsheet calculation**

| WSPR Data (Polar) | | Direction, Θ rad | Plot Coordinates | | Inverted | |
|---|---|---|---|---|---|---|
| | | | x axis | y axis | | |
| Distance, r | Azimuth° | Azimuth° × π/180 | r*cos(Θ) | r*sin(Θ) | x | y |
| 1,501 | 261 | 4.56 | (235) | (1,483) | (1,483) | (235) |
| 1,171 | 82 | 1.43 | 163 | 1,160 | 1,160 | 163 |
| 1,554 | 296 | 5.17 | 681 | (1,397) | (1,397) | 681 |
| 1,469 | 80 | 1.40 | 255 | 1,447 | 1,447 | 255 |
| 2,374 | 74 | 1.29 | 654 | 2,282 | 2,282 | 654 |

In total, I had 528 spots from 115 spotters during 21, two minute transmission sequences over a period of an hour and a half.

## B. Field Test Results

**Figure 10: Total WSPR Spots at 1/8 λ (Low Height) – KM & Azimuth**

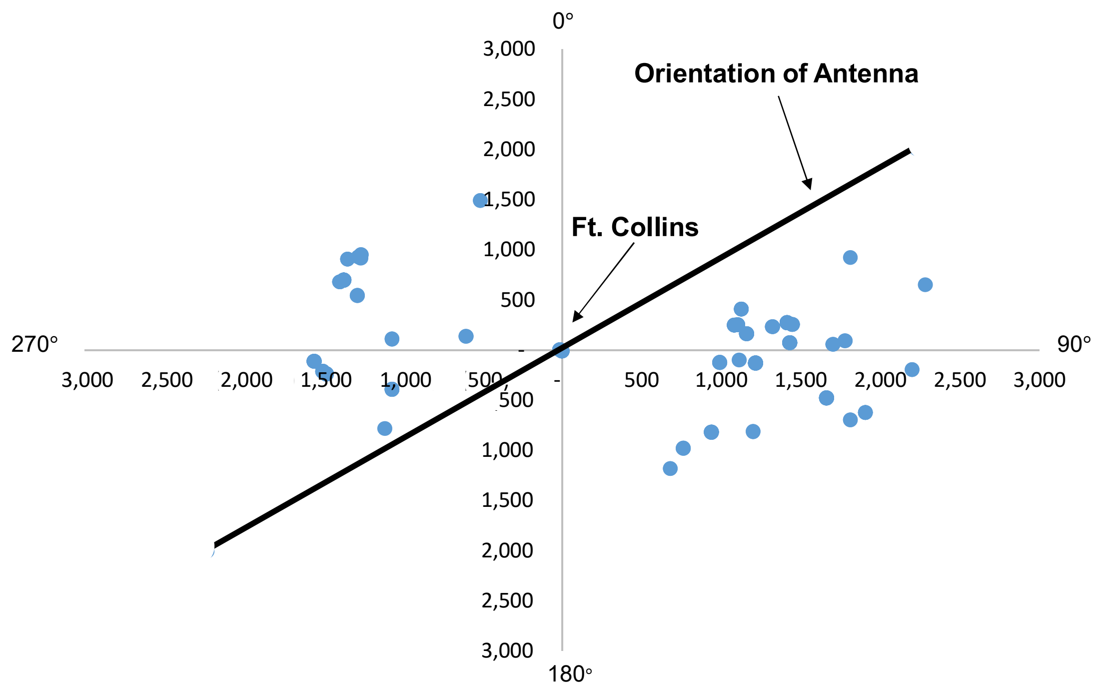



**Figure 11: Total WSPR Spots at 1/3 λ (High Height) – KM & Azimuth**

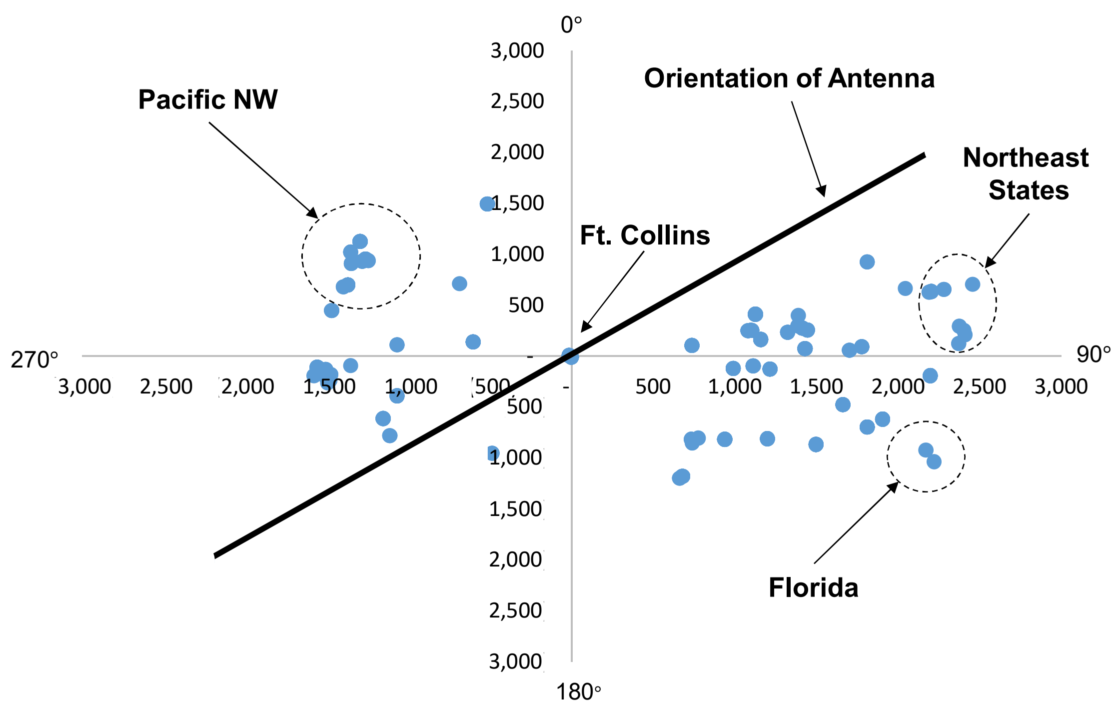


**Table 2: Field test results**

| Distance | 1/8λ Height-Low Spot Count | % | 1/3λ Height-High Spot Count | % |
|---|---|---|---|---|
| 0 - 499 km* | 22 | *12%* | 20 | *6%* |
| 500 - 999 km | 5 | *3%* | 26 | *8%* |
| 1,000 - 1,499 km | 93 | *51%* | 150 | *43%* |
| 1,500 - 1,999 km | 52 | *29%* | 108 | *31%* |
| >= 2,000 km | 10 | *5%* | 42 | *12%* |
| Total Spots | **182** | | **346** | |
| Average Distance km | 1,250 | | 1,419 | |
| WSPR Transmission Sequences | 10 | | 11 | |
| Number of spotters | 43 | | 72 | |
| Average spots per spotter | 4 | | 5 | |
| Average spots per sequence | 18 | | 31 | |

**The near field (<500km) spots were mostly closer than 15km away and were likely a result of ground skip propagation, not sky wave*

# VI. Conclusions

## A. Summary Conclusions

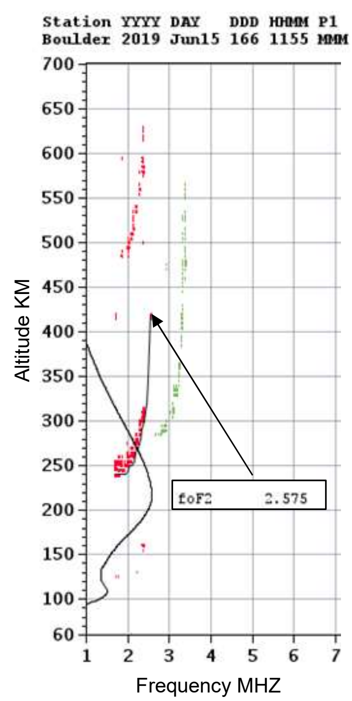
**<u>The results of my experiment are consistent with my hypothesis</u>**. When the dipole was raised:

- The average distance of the spots increased from 1,250km to 1,419km, an increase of 14%.
- The total number of spots increased from 182 to 346, a 90% increase, and the total number of spotters from 43 to 72, a 67% increase.
- The average number of spots per transmission cycle increased by 73% from 18 spots to 31 spots per transmission.

Interestingly, I noticed that raising the antenna resulted in an almost doubling in the overall spot count, while lowering the antenna did not necessarily show more near field spots:

- This may be due partly to the fact that the population density of the East and West Coast is much greater than in a 1,000km radius around Ft. Collins, CO.
- More likely, it is because my signal frequency of 14MHz exceeded the Maximum Usable Frequency (MUF) overhead, and my signals probably passed through the ionosphere for the area directly above my position, limiting spots closer to me. You can see from the vertical incidence ionogram produced from the Boulder, CO iononsonde (approximately 50 miles from my location) that the critical frequency at the time of my test maxed out at 2.575MHz.

**Figure 12: Boulder Ionogram**



# Appendix

## B. Bibliography

[1] Poole, Ian (G3YWX). November 1999 QST: Radio Waves and the Ionosphere. Newington, CT: The American Radio Relay League. 1999.

[2] Nichols, Steve (G0KYA) and Alan Melia (G3NYK). Understanding LF and HF Propagation. Norwich, United Kingdom: Radio Society of Great Britain Propagation Studies Committee, September 2009. Page 14.

[3] The American Radio Relay League. General Class License Manual for Ham radio (Eight Edition): Chapter 7. Newington, CT: The American Radio Relay League, Inc. 2015. Page 7-6.

[4] Straw, Dean R. (N6BV) and Gerald L. Hall (K1TD). Antenna Height and Communications Effectiveness (Second Edition): A Guide for City Planners and Amateur Radio Operators. Newington, CT: The American Radio Relay League, Inc. 1999. Page 7.